

## *Transition properties of $\gamma$ -ray Emitted from levels in $^{90}\text{Mo}$ using $\frac{\sigma}{J}$ method*

*Wafaa Ahmed Azeez, Bashair M.saied, Taghreed A.Younis*

*Department of Physics, University of Zakho*

*Department of physics, Dohok University*

*Department of physics, Baghdad University*

### **Abstract:**

The  $\delta$ -mixing ratios have been calculated for several  $\gamma$ -transitions in  $^{90}\text{Mo}$  using the  $\frac{\sigma}{J}$  method. The results are compared with other references the agreement is found to be very good .this confirms the validity of the  $\frac{\sigma}{J}$  method as a tool for analyzing the angular distribution of  $\gamma$ -ray.

**Key word:** population parameter,  $\gamma$ -ray transition,  $\frac{\sigma}{J}$  method, multiple mixing ratios.

### **Introduction:**

Angular distribution experiment using the reaction  $^{35}_{28}\text{Ni} (^{35}_{17}\text{Cl}, 3p\gamma)^{90}_{42}\text{Mo}$  has been performed at 120 MeV beam energy by kabadiyski et.at [1] . Rasha J.T. calculate the multiple mixing ratios , $\delta$ , of gamma transitions from levels excited in  $^{35}_{28}\text{Ni} (^{35}_{17}\text{Cl}, 3p\gamma)^{90}_{42}\text{Mo}$  by using  $a_2$  -ratio, constant statistical tensor and least square fitting methods .

In the present work, the angular distribution of  $\gamma$ -rays from this reaction are reanalyzed using  $\frac{\sigma}{J}$  method [3]. This method depends on the Gaussian distribution with its half-width  $\sigma$ , where determined using the experimental angular distribution coefficients obtained for a selected number of well-known  $\gamma$ -ray transitions from levels with different spin  $J_i$  Values, the main aim was to confirm the validity of this method as a tool for analyzing the angular distribution of  $\gamma$ -ray.

**Data Reduction & Analysis:**

Yamaszaki [4] has shown that the population parameters of the magnetic sub states of an initial state with spin  $J_i$  and magnetic quantum number  $m_i$  may be represented by a Gaussian distribution of the form:-

$$P(m_i) = \frac{\exp(-m_i^2 / 2\sigma^2)}{\sum_{m=-J_i}^{J_i} \exp(-m_i^2 / 2\sigma^2)} \dots\dots(1)$$

Where  $p(\mathbf{m}_i)$  represents the population parameters and  $\sigma$  is the half-width of the Gaussian distribution.

In the present work, the half-width  $\sigma$ , was determined as follows:

The experimental value of the angular distribution coefficient  $a_2$ , of a certain and well known  $\gamma$ -transition was used to calculate the statistical tensor  $\rho_2(\mathbf{J}_i)$  from the following equation [5]

$$a_2 = \rho_2(\mathbf{J}_i) Q_2 \frac{F_2(L_1L_1J_fJ_i) + 2\delta F_2(L_1L_2J_fJ_i) + \delta^2 F_2(L_2L_2J_fJ_i)}{(1 + \delta^2)} \dots\dots(2)$$

Where  $\delta$  is the multiple mixing ratio  $J_i$  and  $J_f$  are the spin of initial and final states respectively  $L$ , is the angular momentum of  $\gamma$ -ray with  $L_2 = L_1 + 1$  and  $Q$  is the attenuation factor which is considered here to be unity . The  $F_2$ -coefficients are tabulated in ref. [4,5,6] for integer and half – integer  $J$  -values .

The attenuation coefficient,  $\alpha_2 (J_i)$ , was then calculated from the following relation ship [4-6] :-

$$\alpha_2 (J_i) = \rho_2 (J_i) / B_2(J_i) \dots\dots(3)$$

Where  $B_2(J_i)$  is the statistical tensor for the complete alignment and its values are given in

ref. [4]

$$B_k(J) = (2J+1)^{1/2} (-)^J (J_o J_o / K_o) \quad \text{for integer } J \dots\dots(4)$$

And

$$B_k(J) = (2J+1)^{1/2} (-)^{J-1/2} (J_{1/2} J- 1/2 K_o) \quad \text{for half -integer } J \dots\dots(5)$$

The  $\alpha_2 (J_i)$  values are tabulated in ref. [6] for integer values of  $J_i$  from 1 to 26 and half – integer values from 3/2 to 51/2 for  $\frac{\sigma}{J}$  values from 0.1 to 2.0 each  $J_i$  value .from these tables , the half – width ,  $\sigma$  was determined for the  $J_i$  values and was used in eq. (1) to calculate population parameters  $p(mi)$ . The population parameters of levels in  $^{90}\text{Mo}$ ( computed by using computer program in mat lab language) , were it is almost constant for level , with the same  $J_i$  value for both positive and negative parities , it , may, there for , be stated that population parameters of levels with the same  $J_i$  value do not depend upon the energy of the level nor upon its parity . Tacking this fact into consideration, the population parameters thus calculated were used to cover all the possible transitions occurring in the present work.

These population parameters were then, used with statistical tensor coefficients  $\rho_2(J_i, M_i)$  from Ref. [6] in order to calculate statistical Tensor  $\rho_2(J_i)$  from the following equation:

$$\rho_2(J_i) = \sum \rho_2(J_i, m_i) P(m_i) \dots\dots(6)$$

These statistical tensors were then used with  $F_2$  coefficient values to calculate the multipole mixing ratios of  $\gamma$ -transition.

**Result and discussion:-**

- 1- If the differences between  $J_f$  and  $J_i = 2$  and its parity is even, the transition will be pure E2, depending upon this fact the transition ( $7^-5^-$ ) must be pure E2, and this is what we reached it in present work, the  $\delta$ - value for this transition 3367.4 keV ( $7^-5^-$ ) from 818.4 keV level equal (0.04), this  $\delta$ - value is very small even it will be negative or positive by using  $\delta^2 = \frac{M3}{E2}$  when we use,  $|J_i - J_f| \leq L \leq |J_i + J_f|$  the magnetic transition will be odd value and the electric transition will be even value and  $M3+E2 = 100\%$ , this mean that this transition will be 99.8% E2 and (0.04) M3, this indicate that the  $\delta$ - value in present work is accurate and agreement with that in ref[1], [2].

This rule will applied for other transitions:

4192.5keV	( $10^+-8^+$ ) from	1317.7keV	E2=99.999
4555.8 keV	( $12^+-10^+$ ) from	477.0keV	E2=99.998
5699.6 keV	( $13^-11^-$ ) from	857.5keV	E2=99.910
5625.0 keV	( $14^+-12^+$ ) from	1069.1keV	E2=99.990
6643.1 keV	( $15^-13^-$ ) from	943.5keV	E2=99.997
7515.1 keV	( $17^-15^-$ ) from	872.8keV	E2=99.990
8525.4 keV	( $18^+-16^+$ ) from	1779.2keV	E2=99.960
9319.1 keV	( $19^-17^-$ ) from	1804.0ke	E2=99.999

The results of the present work are agreement with other refs [1,2 ] except that the  $(20^+ -18^+)$  transition.

2-If the difference between  $J_i$  and  $J_f =0$  or 1, and have odd parity ,the transition will be pure E1, depending upon this fact the  $(15^- - 14^+)$  transition must be pure E1 this indicated that our  $\delta$ -value , results for 6643.1 keV  $(15^- -14^+)$  from 1018.1 keV are true and by using  $\delta^2 = \frac{M2}{E1}$  ,the magnetic transition must be even and electric transition will be odd and  $E1+M2= 100\%$  this mean that  $E1=99.999\%$  and  $M2=0.002\%$  ,this indicate the  $\delta$ -values present work for this transition are true and in good agreement with that in refs[1,2 ] . and this rules also will be applied for other transition  $(17^- -16^+)$  from 7515.1 keV which have  $E1=99.99\%$  and  $M2= 0.004\%$  ,however for  $(11^- -10^+ )$  4842.1 keV from 649.6 keV and 5699.6 keV  $(13^- -12^+ )$  from 1143.8 keV our results show that this transitions are not pure transition even its they have odd parity .This indicate that the experimental results are inaccurate and this ensured by experimental results from ref[1 ] where presented that this transitions are not E1 and it may be have small  $\delta$ -value.

### **Conclusion:-**

The results of the present work are in very good agreement with those of ref. [1,2] from these comparisons, it may be concluded that the  $\frac{\sigma}{J}$  method is a powerful tool for analyzing angular distributions of  $\gamma$ -ray .it should also be mentioned that the calculations based on the  $\frac{\sigma}{J}$  method can be performed using an ordinary personal computer.

Table (2) *Multipole mixing Ratios of  $\gamma$ - transition from level of  $^{90}\text{MO}$  Using  $\sigma/J$  Method*

$E_i(\text{KeV})$	$E_\gamma(\text{KeV})$	$J_i^{\pi_i} - J_f^{\pi_f}$	$\frac{a_2}{a_4}[\mathbf{1}]$	$\rho_2(J_i)(\text{p.w})$	$\delta$			
					Ref [ 1 ]	CST Method[ 2]	LSF Method[ 2]	$\sigma/J$ Method (p.w)
3367.4	818.4	7-5 <sup>-</sup>	0.26(2) -0.00(2)	-0.5944	E2	0.00(4)	-0.04(2)	0.04(4) ( 6.3 <sup>+1.5</sup> <sub>-1</sub> )
4192.5	1317.7	10 <sup>+</sup> -8 <sup>+</sup>	0.31(1) -0.06(2)	-0.848	E2	0.00(2)	0.00(1)	- (0.003 <sup>+0.007</sup> <sub>-0.01</sub> ) 6.2(4)
4842.1	649.6	11 <sup>-</sup> - 10 <sup>+</sup>	-0.34(30) 0.06(3)	-1.0905	-0.04(6)	0.00(2)	-0.07(2)	-0.33(2) - (7.8 <sup>+1.2</sup> <sub>-0.9</sub> )
4555.8	477.0	12 <sup>+</sup> - 10 <sup>+</sup>	0.33(1) -0.09(2)	-0.9271	E2	0.00(1)	0.00(1)	- (0.004 <sup>+0.006</sup> <sub>-0.01</sub> ) 5.6(3)

5699.6	857.7	13 <sup>-</sup> -11 <sup>-</sup>	0.34(2) -0.09(2)	-1.0513	E2	-0.02(2)	0.01(2)	- ( 0.03 <sup>+0.01</sup> <sub>-0.02</sub> ) ( 6.3 <sup>+0.7</sup> <sub>-0.6</sub> )
	1143.8	13 <sup>-</sup> -12 <sup>+</sup>	-0.28(2) 0.03(2)	-1.0513	-0.02(5)	-0.02(2)	-0.03(1)	- 0.29(1) - ( 10.6 <sup>+1.5</sup> <sub>-1.1</sub> )
5625.0	1069.1	14 <sup>+</sup> -12 <sup>+</sup>	0.35(2) -0.11(33)	-1.0312	E2	0.00(2)	0.01(2)	- ( 0.01 <sup>+0.02</sup> <sub>-0.014</sub> ) ( 5.5 <sup>+0.6</sup> <sub>-0.5</sub> )
6643.1	943.5	15 <sup>-</sup> -13 <sup>-</sup>	0.33(2) 0.04(2)	-0.92332	E2	0.00(2)	-0.01(2)	0.005(0.015) ( 4.9 <sup>+0.5</sup> <sub>-0.4</sub> )
	1018.1	15 <sup>-</sup> -14 <sup>+</sup>	-0.23(2) 0.03(3)	-0.92332	0.01(6)	0.00(2)	0.01(2)	- ( 0.002 <sup>+0.008</sup> <sub>-0.012</sub> ) - ( 12.8 <sup>+2.4</sup> <sub>-1.8</sub> )

7515.1	872.0	17 <sup>-</sup> -15 <sup>-</sup>	0,29(2) -0.05(2)	-0.8003	E2	0.01(3)	-0.05(2)	(0.01 <sup>+0.02</sup> <sub>-0.019</sub> )  4.5(5)
	768..9	17 <sup>-</sup> -16 <sup>+</sup>	-0.19(2) 0.07(2)	-0.8003	0.02(6)	0.01(2)	0.03(1)	(0.004 <sup>+0.016</sup> <sub>-0.014</sub> )  - (14.6 <sup>+3.9</sup> <sub>-2.6</sub> )
8525.4	1779.2	18 <sup>+</sup> -16 <sup>+</sup>	0.33(4) -0.05(4)	-0.89717	E2	0.00(5)	-0.02(3)	(0.02 <sup>+0.01</sup> <sub>-0.04</sub> )  (4.4 <sup>+0.8</sup> <sub>-0.7</sub> )
9319.1	1804.0	19 <sup>-</sup> -17 <sup>-</sup>	0.34(6) -0.02(6)	-0.97766	E2	0.00(7)	-0.01(5)	(0.002 <sup>+0.048</sup> <sub>-0.052</sub> )  (4.6 <sup>+1.4</sup> <sub>-0.9</sub> )
10235.2	1709.9	20 <sup>+</sup> -18 <sup>+</sup>	0.37(3) -0.03(4)	-0.7321	E2	0.00(3)	0.01(3)	0.14(4)  2.7(3)



**Table (1)** *Statistical tensor coefficient, half width and attenuation coefficients for <sup>90</sup>Mo[1,2]*

$E_i(KeV)$	$E_\gamma(KeV)$	$J_i^{\pi_i} - J_f^{\pi_f}$	$\begin{matrix} a_2 \\ a_4 \end{matrix} [1, 2]$	$B_2(J_i)$	$\alpha_2(J_i)$	$\sigma$	$\sigma/J$	$\rho_2(J_i) [2]$	$\rho_2(J_i) p.w$
2548.8	546.7	5 <sup>-</sup> - 4 <sup>+</sup>	$\begin{matrix} -0.20(2) \\ 0.00(2) \end{matrix}$	0	.....	.....	.....	-0.67937(6794)	.....
3367.4	818.6	7 <sup>-</sup> - 5 <sup>-</sup>	$\begin{matrix} 0.26(2) \\ -0.00(2) \end{matrix}$	-1.125604	0.59151	3.15	0.45	-0.66580(5122)	-0.5944
4079.0	972.7	10 <sup>+</sup> - 8 <sup>+</sup>	$\begin{matrix} 0.32(1) \\ -0.082(2) \end{matrix}$	-1.1218632	0.76174	3	0.3	-0.85457(1918)	-0.8480
4192.5	1317.7	10 <sup>+</sup> - 8 <sup>+</sup>	$\begin{matrix} 0.31(1) \\ -0.06(2) \end{matrix}$						
4842.1	649.6	11 <sup>-</sup> - 10 <sup>+</sup>	$\begin{matrix} -0.34(3) \\ 0.06(3) \end{matrix}$	-1.1211954	1.010915	1.1	0.1	-1.13428(11773)	-1.0905
4555.8	476.0	12 <sup>+</sup> - 10 <sup>+</sup>	$\begin{matrix} 0.33(1) \\ -0.09(2) \end{matrix}$	-1.120725	0.81740	3	0.25	-0.91608(2776)	-0.9271
5699.6	1143.8	13 <sup>-</sup> - 12 <sup>+</sup>	$\begin{matrix} -0.28(2) \\ 0.03(2) \end{matrix}$	-1.120380	0.89931	1.95	0.15	-1.00757(4589)	-1.0513
	857.7	13 <sup>-</sup> - 11 <sup>-</sup>	$\begin{matrix} 0.34(2) \\ -0.09(2) \end{matrix}$	-1.120354	0.89933	3.25	0.25		-0.9259
5625.0	1069.1	14 <sup>+</sup> - 12 <sup>+</sup>	$\begin{matrix} 0.35(2) \\ -0.11(3) \end{matrix}$	-1.120039	0.881987	2.8	0.2	-0.98786(5645)	-1.0312

<b>6643.1</b>	<b>1018.1</b> <b>943.5</b>	<b>15<sup>-</sup> -14<sup>+</sup></b> <b>15<sup>-</sup> - 13<sup>-</sup></b>	<b>-0.23(2)</b> <b>0.03(3)</b> <b>0.33(2)</b> <b>0.04(2)</b>	<b>-1.119749</b>	<b>0.836220</b> <b>0.83622</b>	<b>3.75</b>	<b>0.25</b>	<b>-093635(4656)</b>	<b>-0.92332499</b>
<b>7515.1</b>	<b>872.0</b> <b>768.9</b>	<b>17<sup>-</sup> -15<sup>-</sup></b> <b>17<sup>-</sup> -16<sup>+</sup></b>	<b>0.29(2)</b> <b>-0.05(2)</b> <b>-0.19(2)</b> <b>0.07(2)</b>	<b>-1.119408</b> <b>-1.119375</b>	<b>0.728658</b> <b>0.728677</b>	<b>5.1</b>	<b>0.3</b>	<b>-0.81566(4707)</b>	<b>-0.8003</b>

**Table (1) cont.**

<b>8525.4</b>	<b>1779.2</b>	<b>18<sup>+</sup> -16<sup>+</sup></b>	<b>0.33(4)</b> <b>-0.05(4)</b>	<b>-1.119270</b>	<b>0.851045</b>	<b>4.5</b>	<b>0.25</b>	<b>-0.95255(11546)</b>	<b>-08971702</b>
<b>9319.1</b>	<b>1804.0</b>	<b>19<sup>-</sup> -17<sup>-</sup></b>	<b>0.34(6)</b> <b>-0.02(6)</b>	<b>-1.119130</b>	<b>0.88060</b>	<b>3.8</b>	<b>0.2</b>	<b>-0.98551(17391)</b>	<b>0.9776691</b>
<b>10235.2</b>	<b>1709.9</b>	<b>20<sup>+</sup> -18<sup>+</sup></b>	<b>0.37(3)</b> <b>-0.03(4)</b>	<b>-1.119057</b>	<b>0.961988</b>	<b>2</b>	<b>0.1</b>	<b>-1.07652(8729)</b>	<b>-0.7321</b>

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